Asynchronous Decentralized Algorithm for Space-Time Cooperative Pathfinding

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Abstract. Cooperative pathfinding is a multi-agent path planning problem where a group of vehicles searches for a corresponding set of non-conflicting space-time trajectories. Many of the practical methods for centralized solving of cooperative pathfinding problems are based on the prioritized planning strategy. However, in some domains (e.g., multi-robot teams of unmanned aerial vehicles, autonomous underwater vehicles, or unmanned ground vehicles) a decentralized approach may be more desirable than a centralized one due to communication limitations imposed by the domain and/or privacy concerns.

In this paper we present an asynchronous decentralized variant of prioritized planning ADPP and its interruptible version IADPP. The algorithm exploits the inherent parallelism of distributed systems and allows for a speed up of the computation process. Unlike the synchronized planning approaches, the algorithm allows an agent to react to updates about other agents' paths *immediately* and invoke its local spatio-temporal path planner to find the best trajectory, as response to the other agents' choices. We provide a proof of correctness of the algorithms and experimentally evaluate them on synthetic domains.

1 Introduction

When mobile agents operate in a shared space, one of the essential tasks for them is to prevent collisions among themselves, possibly even to maintain a safe distance from each other. Prominent examples of domains requiring robust collision avoidance techniques are different kinds of autonomous multi-robotic systems, next-generation air traffic management systems, road traffic management systems etc.

A range of methods is being currently employed to realize a safe operation of agents within a shared space. Some of the methods assume a cooperative setting where all the involved agents work together to solve their mutual conflicts, others assume a non-cooperative setting where the agents cannot coordinate their actions, and yet others consider pursuit-evasion adversarial scenarios where a solution is a trajectory that is collision free against the worst-case behaviour of other agents. In this work, we focus on the cooperative pathfinding.

Cooperative path planners are used to plan the routes for a number of agents, taking in consideration the objectives of each agent while avoiding conflicts between the agents' paths. If the agents execute the resulting multi-agent plan precisely, it is guaranteed that the agents will not collide. Centralized solvers in literature are based either on global search or decoupled planning. Global search methods find optimal solutions, but they do not scale well for higher (over ten) numbers of conflicting agents. One of the most efficient optimal solvers for cooperative pathfinding on grids has been introduced by Standley in 2010 [8].

Decoupled approaches are incomplete, but can be fast enough for real-time applications e.g., in the video-game industry. One of the the standard technique employed in gaming is the *Local Repair A** (LCA*) algorithm [7]. In LCA* each agent plans a path independently and tries to follow it to the goal position. If a collision occurs during the path plan execution, the agent replans the remainder of the route from the collision position taking into account the positions in its vicinity occupied by the other agents involved in the collision. Due to its greedy and reactive nature, the method does not perform well in cluttered environments with bottlenecks and can generate cycles, or otherwise aesthetically unpleasant, or inefficient behaviours of agents [6]. To mitigate these problems, Silver [7] introduced *Cooperative A** (CA) a cooperative pathfinding algorithm based on the idea of prioritized planning [3].

In prioritized planing, each agent is assigned a priority and the planning process proceeds sequentially agent after agent in the order of the agents' priorities. The first agent plans its path using a single-agent planner disregarding the positions and objectives of other agents. Each subsequent agent models the paths of the higher-priority agents as moving obstacles and plans its path such that the collisions with the higher-priority agents' paths are avoided. Such an approach has been shown to be effective in practice [4]. The quality of the generated solution is sensitive to the assigned priority ordering, however there is a simple heuristic for choosing an efficient ordering for the prioritized planning [9].

Recently, Velagapudi presented a decentralized prioritized planning technique for large teams of mobile robots [10]. The method is shown to generate the same results as the centralized planner. However, the formulation of the decentralized algorithm is based on the assumption that the robots have a "distributed synchronization mechanism allowing them to wait for all team mates to reach a certain point in algorithm execution" [10] and thus it does not exploit the asynchrony common in distributed systems. Rather the computation proceeds in iterations and the agents wait for each other at the end of each algorithm iteration. As a consequence, the algorithm does proceeds in synchronous rounds, where the length of a round is dictated by the agent performing the longest computation due to either a high workload, or low computational resources available.

After stating the cooperative pathfinding problem and exposing the underlying ideas of the state-of-the-art prioritized planning approaches in Sections 2 and 3, Section 4 presents the main contribution of the paper, the *asynchronous decentralized prioritized planning algorithm* (ADPP). ADPP, is an extension of the synchronized

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decentralized prioritized planning algorithm (SDPP), which removes the assumptions of synchronous execution of the decentralized algorithm. Besides the generic form of the ADPP algorithm, we also present a locally asynchronous modification of the ADPP algorithm (*interruptible ADPP*, IADPP) enabling interruptible path planner execution so that the individual agents can react to updates received from their peers more swiftly. To prove termination and correctness properties of ADPP and IADPP, we provide a new proof of termination and correctness also for the SDPP algorithm. Our proof is an alternative to the original argument presented in [10]. We implemented and extensively evaluated the discussed algorithms on a number of synthetic scenarios. Section 5 provides both an illustrative theoretical comparison of the SDPP and ADPP approaches, as well as details the experimental evaluation of the introduced algorithms. The experimental validations show that the asynchronous versions of the prioritized planning algorithm offer better runtime performance, as well as improved use of the available computational resources.

2 Cooperative Pathfinding Problem

Consider n agents $a_1,\ldots,a_n$ operating in an Euclidean space $\mathscr{W}$. Each agent a_i is characterized by its starting and goal positions $start_i$, $dest_i$ respectively. The task is to find a set of space-time trajectories $P=\{p_1,\ldots,p_n\}$, such that $p_i:\mathbb{R}\rightarrow\mathscr{W}$ is a mapping from time points to positions in $\mathscr{W}$, $p_i(0)=start_i$, $p_i(t_i)=dest_i$ and the trajectories are mutually collision free, i.e., $\forall i,j: i\neq j\Rightarrow\neg C(p_i,p_j)$, where $C(p_i,p_j)$ denotes a space-time mutual collision relation between p_i and p_j. Informally, two trajectories collide (are in a conflict) when the trajectories touch, or intersect. That is $p_i[t'] = p_j[t']$ for some timepoint t'. However, more complex collision relations can be considered, such as those considering a minimal separation range between trajectories, etc. $t_i^{dest}=\min\{t_i \mid p_i[t_i]=g_i\}$ denotes the shortest timepoint in which the agent a_i reaches its destination $dest_i$. As a solution quality metric we use the cumulative time spent by agents navigating their trajectories defined as $dur(P)=\sum_{i=1}^{n}t_i^{dest}$. The cost of solution P is defined as $cost(P)=\frac{dur(P)-dur(P')}{dur(P')}$, where P' is the set of best trajectories for each agent if the collisions are ignored.

3 Prioritized Planning

In general, the complexity of complete approaches to multi-agent path planning grows exponentially with the number of agents. Therefore, the complete approaches often do not scale-up well and hence are often not applicable for nontrivial domains with many agents. To plan paths for a high number of agents in a complex environment, one has to resort to one of the incomplete, but fast approaches. A simple method often used in practice is *prioritized* planning [3, 9, 1]. In prioritized planning the agents are assigned a unique priority. In its simplest form, the algorithm proceeds sequentially and agents plan individually from the highest priority agent to the lowest one. The agents consider the trajectories of higher priority agents as constraints (moving obstacles), which they need to avoid. It is straightforward to see that when the algorithm finishes, each agent is assigned a trajectory not colliding with either higher priority agents, since the agent avoided a collision with those, nor with lower priority agents who avoided a conflict with the given trajectory themselves.

The complexity of the generic algorithm grows linearly with the number of agents, which makes the approach applicable for problems involving many agents. Clearly, the algorithm is greedy and incomplete in the sense that agents are satisfied with the first trajectory not colliding with higher priority agents and if a single agent is unable to find a collision-free path for itself, the overall path finding algorithm fails. The benefit, however, is fast runtime in relatively uncluttered environments, which is often the case in multi-robotic applications. Prioritized planner is also sensitive to the initial prioritization of the agents. Both phenomena are illustrated in Figure 1 that shows a simple scenario with two agents desiring to move from s_1 to d_1 (s_2 to d_2 resp.) in a corridor that is only slightly wider than a single agent. The scenario assumes that both agents have identical maximum speeds.

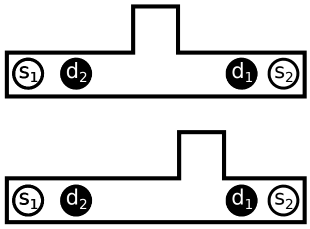


Figure 1: Top: example of a problem to which a prioritized planner will not find a solution. The first agent plans its optimal path first, but such a trajectory is in conflict with all feasible trajectories of the second agent. Bottom: example of a problem to which a prioritized planner will find a solution only if agent 1 has a higher priority than agent 2.

3.1 Computing best response

During prioritized planning, an individual agent searches the shortest path to its destination considering other higher-priority agents as moving obstacles during the planning process. Ideally, the agent should compute the best possible trajectory, a best response to the trajectories of the higher-priority agents. To find such a best response, the agent needs to solve a motion planning problem with dynamic obstacles, which is a significantly more complex task than the motion planning with static obstacles since a new independent time dimension has to be considered during planning. Henceforth, we will denote the single-agent best-response planer process as a function $\textsc{Best-path}_i(start, dest, avoids)$, which returns the selected best trajectory for the agent i, starting in the position *start*, eventually reaching the position *dest* and at the same time not colliding with any of the trajectories in the set *avoids*. Note, we do not precisely specify what the best trajectory means, the notion can be application-specific for the individual agent. For simplicity, however, in the following we assume the notion of the best path to correlate with time-optimality of trajectories, i.e., the how fast a given agent can navigate along the trajectory given its specific motion dynamic constraints.

3.2 Centralized Algorithm

A collision-free operation of a multi-agent team can be ensured by forcing all agents to communicate their objectives to a centralized planner, which centrally computes a solution and informs the agents about the trajectory they have to follow in order to maintain the conflict-free operation. As a baseline for evaluation of performance of the latter introduced algorithms, we use the *cooperative A* algorithm* [7]. Cooperative A* is a centralized algorithm for cooperative path finding based on prioritized planning employing the well-known A* trajectory planning algorithm on grids [5]. Algorithm 1 lists the pseudocode of the cooperative A* algorithm. We discussed the correctness of this generic algorithm above.

Algorithm 1 Centralized Prioritized Planning (Cooperative A*)

Ensure: After the algorithm finishes, $Path_i$ contains the final computed path for the agent with priority i. If the agent couldn't find a path not colliding with higher priority agents, $Path_i$ stores $\emptyset$.

```
procedure CA(⟨start_1, dest_1⟩, ..., ⟨start_n, dest_n⟩)
    Avoids ← ∅
    for i ← 1...n do
        Path_i ← BEST-PATH_i(start_i, dest_i, Avoids)
        Avoids ← Avoids ∪ {Path_i}
    end for
end procedure

function BEST-PATH_i(start, dest, avoids)
    return the best path from start to dest not conflicting with
            any of the paths in avoids. Otherwise return ∅.
end function
```

Algorithm 2 Synchronized Decentralized Prioritized Planning
▷ *pseudocode for the agent i* ◁

Ensure: After the algorithm finishes, $Path_i$ contains the final computed path. If no solution was found, $Path_i$ stores $\emptyset$.

```
procedure SDPP(start, dest, nagents, priority)
    Start_i ← start; Dest_i ← dest
    N ← nagents; I ← priority
    Agentview_i ← ∅; Path_i ← ∅
    repeat
        CHECK-CONSISTENCY-AND-PLAN
        wait for all other agents to finish the planning iteration
    until global termination detected
end procedure

procedure CHECK-CONSISTENCY-AND-PLAN
    if Path_i collides with Agentview_i then
        ▷ Work on a copy of the Agentview_i ◁
        Path_i ← BEST-PATH_i(Start_i, Dest_i, Agentview_i)
        for all j ← I+1...N do
            SEND-INFORM-TO-j(I, Path_i)
        end for
    end if
end procedure

message handler RECEIVE-INFORM(j, path)
    Agentview_i ← (Agentview_i \ ⟨j, _⟩) ∪ {⟨j, path⟩}
end message handler
```

3.3 Decentralized Algorithms

A decentralized algorithm for solving cooperative pathfinding problems by means of prioritized planning has been presented in [10]. The algorithm is synchronous in that it contains synchronization points in the program execution through which all agents proceed simultaneously. Due to the synchronous nature of the algorithm, we will refer to this algorithm as *synchronized decentralized prioritized planning* (SDPP). Algorithm 2 lists the pseudocode of SDPP. We slightly adapted the algorithm listing for exposition purposes and comparison with the later introduced algorithms. Note that in the decentralized setting we assume communication to be reliable and the communication channel preserves the order of messages they were sent in. Furthermore, the algorithm assumes that before the start of the algorithm, each agent is assigned a unique priority, an ordinal $I \in 1 \dots N$, where N is the number of agents taking part on the algorithm run (the lowest I means the highest priority). The algorithm is also locally asynchronous and we assume safe (thread-safe) access to global variables (denoted by capitalized identifiers). To simplify the exposure, the thread-barrier locking mechanism is omitted from the pseudocode.

The algorithm proceeds in iterations. In each iteration the agents compute the best path if necessary and subsequently communicate it to the lower priority agents. An agent must recompute its trajectory in the case its current path collides with some trajectories of higher priority agents computed and communicated in the previous iterations. Upon receiving an INFORM message, the agent simply replaces the information about the trajectory of the sender agent in its $Agentview_i$ set. Note, the algorithm is asynchronous, therefore the trajectory planning routine BEST-PATH$_i$ operates on a copy the $Agentview_i$ set.

The algorithm finishes when all the agents cease to communicate and either hold a trajectory, or they were not able to find a collision-free trajectory. We assume that the global termination condition is detected by some concurrently running global state detection algorithm, such as the Chandy and Lamport's snapshot algorithm [2].

The presented SDPP algorithm is correct in that when it finds a solution for all the participating agents, the paths are mutually collision free. However, the algorithm is incomplete in the sense that there are situations when the algorithm fails to find a solution for all the participating agents, even though such a solution exists. In order to facilitate and simplify exposition of the later introduced algorithms, we developed a new alternative proof of the SDPP algorithm, which deviates from the original one devised by the authors of SDPP **[10]**.

To see the correctness of the SDPP algorithm we need to show that firstly, the algorithm terminates, and secondly that the resulting paths are mutually collision free.

Proof (SDPP termination):. First of all, we need to show that the algorithm finishes. That is, each agent i eventually stops sending INFORM messages. We proceed by induction on the individual agent priority i.

initial step since there is no agent with priority higher than agent a_1, the highest priority agent a_1 informs the lower priority agents only once in the first iteration of the algorithm and from then on it remains silent since its path will always be non-colliding with an empty set of paths - there are no higher priority agents to inform this agent about an update of the situation.

induction step Let's assume the following induction hypothesis: *"after the agents with priorities $1 \dots k-1$ stopped communicating, eventually also the agent with priority k stops sending* INFORM *messages"*. Let's assume this is not the case and there is a situation such that the agent k would end up sending INFORM messages forever. For such to occur, the agent however must have its mailbox continually being filled with INFORM messages so that it's RECEIVE-INFORM handler routine gets invoked infinitely many times. In a consequence the agent would possibly need to recompute its best path and subsequently inform the lower priority agents infinitely often. That however implies existence of a sender for each such a message and hence by necessity there must be at least one agent with priority higher than k which keeps sending INFORM messages forever, which contradicts the induction hypothesis.

□

As a consequence of the consecutive silencing of agents from high to lower priorities, it's also relatively straightforward to see that the SDPP algorithm makes at most N iterations before it terminates.

Note, that not necessarily it is the agent with the lowest priority which stops communicating the last. In the case a lower priority agent computes a route which is not in a conflict with a current set of temporary routes of the higher priority agents, nor with any routes they will compute later on, its reactions to receiving INFORM messages will be silent and won't result in further cascade of communication.

Proof (SDPP correctness):. To see that after the algorithm termination the variables $Path_i$ store a set of non-conflicting paths is rather straightforward. Since each agent eventually sends its last INFORM message and cedes to communicate, each agent with priority lower than its own eventually collects all the last INFORM messages from all the higher priority agents, together with their ultimate paths (being either a valid path, or $\emptyset$). At that moment, all the couples $\langle j, Path_j \rangle$ for all $j > i$ are stored in the set $Agentview_i$ of the agent with priority i. Subsequently the agent eventually invokes the CHECK-CONSISTENCY-AND-PLAN routine for the last time and thus either $Path_i$ will end up unchanged, recomputed and again non-conflicting with either of $\langle j, Path_j \rangle$ for all $j < i$, or being invalid ($\emptyset$). Finally, the agent informs all the agents with priorities lower than i and cedes to communicate. At the moment when the last agent stops communicating, all the $Path_i$ variables are either set correctly, or the algorithm failed to find a solution for some of the participating agents. □

As we already noted above, the SDPP algorithm is incomplete. To see that, consider a situation in which the agent with the highest priority makes a choice which later on constraints some of the lower priority agents so that they are unable to find a solution. In the case there would be a locally worse choice for the highest priority agent, which however would enable the lower priority agents to find valid solutions, the SDPP algorithm does not facilitate re-consideration of the first choice, nor some backtracking mechanism.

During the algorithm computation, it can however happen that an agent i sets its $Path_i$ to $\emptyset$ and later reconsiders this decision. This happens when among paths of the higher priority agents there are conflicting couples, but those agents did not manage to resolve the collisions yet and at the same time the lower priority agent i is temporarily not able to route around the space occupied by the temporary paths of the higher priority agents.

Note that in the distributed prioritized planning, one can use a simple marking-based termination-detection mechanism. Following the proof of termination, agent i can mark its path *final* if the path of agent priority $i-1$ in $Agentview_i$ is marked final. The initial path of a_1 is final. When an agent sends his final path to a lower-priority agent, the higher-priority agent can safely terminate its computation. When the final path is generated by the lowest-priority agent, the computation terminated globally.

4 Asynchronous Prioritized Planning

The SDPP algorithm does not fully exploit the parallelism of the distributed system, a drawback stemming from its synchronous nature. The running time of a single iteration of the SDPP algorithm is largely influenced by the speed of the computationally slowest agent of the group. In every iteration, the agents which finished their trajectory planning routine faster, or did not have to re-plan at all sit idle while waiting for the agents with higher workload in that iteration (or simply slower computation), even though they could theoretically resolve some of the conflicts they have among themselves in the meantime and thus speed up the overall algorithm run.

Algorithm 3 Asynchronous Decentralized Prioritized Planning
▷ *pseudocode for the agent i* ◁

procedure ADPP($start, dest, nagents, priority$)
    $Start_i \leftarrow start$; $Dest_i \leftarrow dest$
    $N \leftarrow nagents$; $I \leftarrow priority$
    $Agentview_i \leftarrow \emptyset$; $Path_i \leftarrow \emptyset$
    repeat
        $CheckFlag_i \leftarrow false$
        CHECK-CONSISTENCY-AND-PLAN
        wait for $CheckFlag_i$, or global termination
    until global termination detected
end procedure

message handler RECEIVE-INFORM($j, path$)
    $Agentview_i \leftarrow (Agentview_i \setminus \langle j, \_ \rangle) \cup \{\langle j, path \rangle\}$
    $CheckFlag_i \leftarrow true$
end message handler

To improve the performance of the decentralized cooperative path finding, we propose an *asynchronous decentralized prioritized planning algorithm* (ADPP), an asynchronous variant of SDPP. Algorithm 3 lists the pseudocode of ADPP.

The main deviation from the SDPP listed in Algorithm 2 is the formulation of the waiting condition in the main loop of the algorithm. While each agent of the group waits for all the other to finish in the SDPP algorithm, in the ADPP algorithm, they break their idle upon receiving the next INFORM message or a need to process updated $Agentview_i$, in the case the agent received a number of INFORM messages during the time it was occupied with planning its own trajectory. The arrival of a new INFORM message and thus the need to re-check the consistency of the currently computed path with respect to the new information is indicated by the state of the $CheckFlag_i$ variable.

The proof of correctness of the ADPP algorithm follows exactly the correctness proof of the SDPP algorithm above. Note, in the SDPP proof, the condition that the algorithm proceeds in a synchronized manner was never exploited. The ADPP algorithm terminates for exactly the same reasons as SDPP. Namely, the agent with the highest priority stops communicating right after it computes its path for the first time and in consequence the agents with lower priority consecutively cede to communicate later on as well until the algorithm terminates. The argument for ADPP incompleteness follows the incompleteness argument for SDPP as well.

Interruptible ADPP

The ADPP algorithm exploits the potential speed up with respect to the inter-agent communication. However, while the agent is computing the best path in the current situation, messages keep arriving. In a consequence, it can happen that an individual agent's computation returns from the path planning routine BEST-PATH$_i$ only to find out that large part of the work was invalidated by some later received messages. This reveals a potential further speed-up of the ADPP algorithm by interrupting the path planning upon reception of every INFORM message and re-considering the computation in the light of the newly received message. Algorithm 4 lists a pseudocode of a modified ADPP algorithm which pro-actively interrupts the trajectory planning computation upon receiving every new INFORM message. Alternatively, it is conceivable to exploit algorithms

Algorithm 4 Interruptible Asynchronous Decentralized Prioritized Planning - pseudocode for the agent i

procedure IADPP($start, dest, nagents, priority$)
  $Start_i \leftarrow start$; $Dest_i \leftarrow dest$
  $N \leftarrow nagents$; $I \leftarrow priority$
  $Agentview_i \leftarrow \emptyset$; $Path_i \leftarrow \emptyset$
  CHECK-CONSISTENCY-AND-PLAN
  wait for global termination
end procedure

message handler RECEIVE-INFORM($j, path$)
  $Agentview_i \leftarrow (Agentview_i \setminus \langle j, \_\rangle) \cup \{\langle j, path\rangle\}$
  asynchronously launch/restart {
    CHECK-CONSISTENCY-AND-PLAN}
end message handler

for dynamic trajectory planning, which allow topological changes during the planning process.

Note, the main **repeat-until** loop was replaced by simple wait for the algorithm termination. The repeated consistency check (calls of the CHECK-CONSISTENCY-AND-PLAN routine) is secured by its asynchronous invocation from the RECEIVE-INFORM routine. That is, the routine is executed in a newly created computation run (thread) and the call does not wait for its termination, it runs in parallel to the RECEIVE-INFORM routine from then on. In the case there is already a concurrent invocation of the CHECK-CONSISTENCY-AND-PLAN routine running, it is killed and run anew (restarted) with the updated $Agentview_i$ set.

The termination and correctness of the IADPP algorithm stems from the termination and correctness of the ADPP algorithm. The same proof applies, since the IADPP modification was strictly local, not affecting the communication patterns between the participating agents.

5 Evaluation

The motivation for introducing the decentralized algorithm and its asynchronous variants is oriented mainly to the runtime improvements of the algorithm. Clearly, such a potential improvement is greatly influenced by the topology of the problem and the selection of agent priorities. In this section, we first discuss the noticeable features of the presented algorithms and our expectation on their performance. Then, we will present experimental evaluations using superconflict and randomly generated scenarios.

5.1 Theoretical analysis

As indicated above, the decentralized approaches should benefit from the concurrent execution on a higher number of processors (i.e., equal, or higher than the number of agents). The wall-clock runtime of the algorithms is expected to be lower for decentralized algorithms, but there might exist some problem configurations that yield directly opposite results. In this section we sketch a theoretical analysis of the impact of the parallelism and asynchronicity and show examples to demonstrate the presented ideas.

Let us first discuss the differences between the centralized and decentralized approaches. For simplicity, let the processing time of the best-path search routine be one time unit for each path searched (one path for one agent). Figure 2 illustrates an example of the algorithm execution sequence for three agents, where priorities of the agents are given from left to right and match the agent indices. The centralized algorithm simply computes the agents' paths sequentially in the order of agents' priorities. The total wall-clock runtime is 3 time units here.

To analyze the algorithm runs in decentralized scenarios, consider a scenario where the agents have non-conflicting trajectories and a superconflict scenario, in which the best trajectories of all the agents collide. In a distributed setting, we assume three parallel processors, i.e. one for each agent. In the case of non-conflicting trajectories the agents should be able to fully utilize the inherent parallelism of the distributed system, so that the wall-clock runtime of the algorithm is only one time unit. However, in the case of superconflict scenario the situation is different. Each lower-priority agent has to recompute his path when a higher-priority agent produces a new solution. Clearly, the parallel execution has no speed-up effect here since the wall-clock runtime stays 3 time units. This example provides an intuition for the bounds of the decentralized algorithm execution time. One would expect that the wall-clock runtime of a decentralized algorithm will be equal or lower than the execution time of the centralized algorithm depending on the scale of coupling between the agents. That is, informally, on the size of a cluster in which agents' trajectories influence each other.

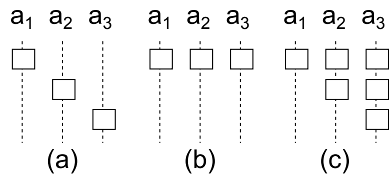


Figure 2: Example of the path search execution sequence for (a) centralized algorithm, (b) decentralized algorithm for non-colliding trajectories problem and (c) decentralized algorithm for mutually-colliding trajectories problem. The boxes represent invocations of best-response planners.

However, the situation changes if we assume non-uniform runtimes of the agents' best-response planers. In such a situation, SDPP may suffer from significant synchronization overheads. Figure 3 illustrates the difference between the synchronous and the asynchronous variant of the decentralized approach. In this example ADPP exploits existence of independent conflict clusters and is able to lower the total wall-clock runtime from 5 to 4 time units. Main distinguishing feature of the ADPP algorithm over SDPP is that in ADPP an agent starts resolving conflicts immediately after the agent detects them, while in SDPP the conflicts are resolved in the next iteration of the algorithm. Since the duration of one SDPP iteration is determined by the slowest computing agent, the computational power of faster computing agents may stay unutilized. This example illustrates how can be the wall-clock runtime reduced by the asynchronous algorithm.

The interruptible variant of ADPP strengthens the asynchronous aspect of the ADPP. Figure 4 shows a another example of the decentralized algorithms execution sequence. The total running time is 5 time units for SDPP and ADPP while IADPP is able to shorten the execution to 4 time units.

5.2 Experimental evaluation

We compare the centralized CA, SDPP, ADPP and IADPP on a few variants of superconflict scenario and on a series of randomly generated problem instances. The experiments were performed on Intel

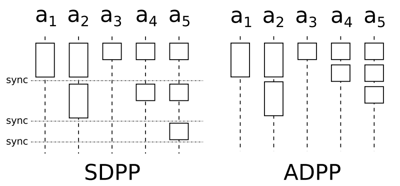


Figure 3: Sequence diagram showing the execution of SDPP resolution process and ADPP resolution process for a scenario with two independent conflict clusters, where agents in $\{a_1, a_2\}$ and $\{a_3, a_4, a_5\}$ need different amount of time to find their best response.

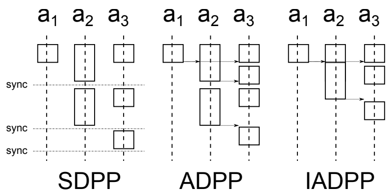


Figure 4: Sequence diagram illustrating how can be wall-clock runtime further reduced by interrupting the best-response planning.

Core 2 Duo @ 2.1 Ghz. The problem instances used have the following common structure. A given number of agents n operate in a shared 20 m x 20 m 2-d square space. The agents generate a space-time trajectory between their start and the destination position using a 4- or 8- connected grid graph. The agents can move on the edges of the graph with the constant speed of 1 m/s or they can wait for 0.5 s on any of the vertices in the graph. The wait "move" can be used repeatedly. The agents are required to maintain the separation distance 0.8 m from all other agents at all times, even after they reached their destination.

The best-response planner used by all the agents is a spatio-temporal A* planner operating over the grid graph, where the heuristic is the time needed to travel the euclidean distance from the current node to the destination node at the maximum speed. All the compared algorithms use the identical best-response planner.

To measure the runtime characteristics of the execution of decentralized algorithms, we emulate the concurrent execution of the algorithms using a discrete-event simulation. The simulation measures the execution time of each message handling and uses the information to simulate the concurrent execution of the decentralized algorithm as if it is executed on n independent computers. In the simulation we assume zero communication delay. The concurrent process execution simulator was implemented using Alite multi-agent simulation toolkit. The complete source code of the experimental environment (including the concurrent process simulator) and the video recordings of the experiments are available at `http://labe.felk.cvut.cz/~mcap/adpp/`.

Superconflict scenarios

We performed a number of experiments on a few variants of a challenging superfconflict scenario. In the superconflict scenario, the agents' start positions are put evenly spaced on a circle and their goal positions are exactly at the opposite side of the circle. Therefore, the agents' nominal trajectories all cross in the center of the circle. The superconflict scenario is considered a challenging benchmark since each agent participating in one superconflict circle is in conflict with all other agents of that circle. Due to this coupling, the problem cannot be easily split into independent subproblems and solved in parallel. In our implementation, the agents plan their trajectory using a 60x60 8-connected grid graph. We evaluated the algorithms on the following variants of superconflict scenario:

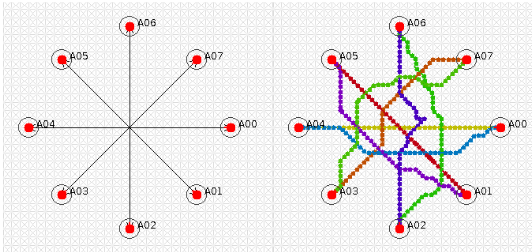
Single supercoflict scenario with a 4 meters-wide superconflict of 8 agents placed in the middle of the square space. Agents' starting configuration and the final trajectories obtained from IADPP are depicted in Figure 5a. Note that $A00$ is the highest priority agent in all our experiments.

Four homogeneous superconflicts scenario with four independent superconflicts of 8 agents (4 meters wide). This scenario allows the cooperative pathfinding problem to be split into four independent parts and thus the decentralized algorithms have an opportunity to exploit the computational power of more processor (see Figure 5b).

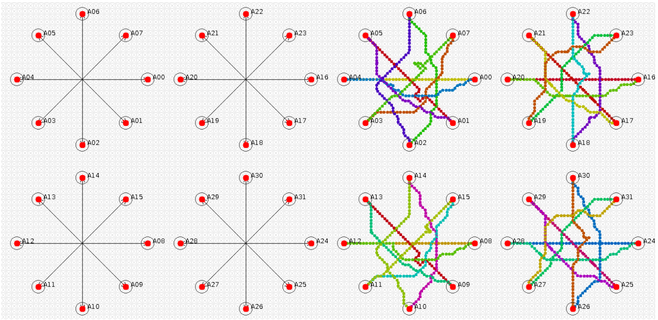
Four heterogeneous superconflicts scenario that combines two superconflicts of four agents (4 meters wide) and two superconflicts of eight agents (only 2 meters wide). The former two have bigger radius than the latter two and thus we expect that the best-response planner invocations in the first group of superconflicts will take on average longer to finish than the planners of the agents from the second group. Such a difference in planning times leads to an inefficient execution of SDPP, since the slowest progressing cluster of conflicts limits the speed at which the other conflict clusters are resolved. The asynchronous algorithm can resolve each of the superconflicts at a different pace and thus we expect ADPP and IADPP to converge faster than SDPP (see Figure 5c).

Spiral superconflict scenario is a superconflict of eight agents, where the distance between an agent's start position and the center of the superconflict increases with each agent. In our scenario the radius varies between 2 m and 6 m. In result, the higher priority agents often finish planning before the lower priority agents and since all the agents are in mutual conflict, the planning process of the lower priority agents is often invalidated. In both SDPP and ADPP, the planning cannot be interrupted, and the agent will adapt to the new situation only after the currently running planning process finishes. Since the interruptible version of ADPP is designed to mitigate this problem, we expect that it will outperform the other decentralized methods in the scenario (see Figure 5d).

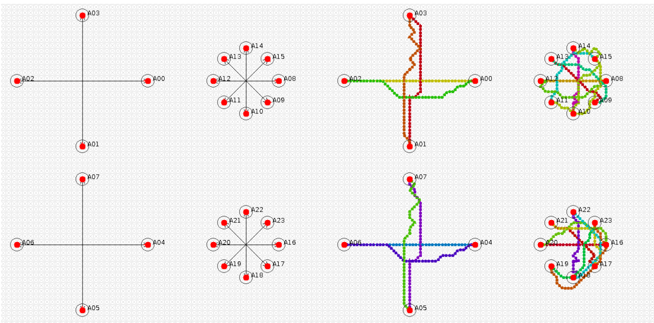
Table 1 shows the wall-clock runtimes of the four evaluated algorithms in the four presented scenarios. For the single superconflict scenario, ADPP and IADPP runtimes are close to CA, but SDPP shows significant synchronization overheads. The second scenario in fact contains four independent instances of the single superconflict as used in the first scenario. The total complexity of this problem is expected to be four times higher than that of the first scenario. The runtime of CA is more than quadrupled, while the runtime of the decentralized algorithms stays almost unchanged, which indicates perfect parallelization of the solution search process. In the heterogeneous variant of the last scenario, the situations looks different. As we can see from CA, the total complexity of the problem is slightly lower than that of the first scenario. Due to the differences in average planning times in the individual superconflicts, the wall-clock runtime in SDPP is dominated by the slowest progressing superconflict. We can see that both ADPP and IADPP can handle the heterogeneity well. The spiral superconflict is a challenging scenario for the non-interruptible asynchronous method. Thus, the ADPP wall-clock runtime is closer to that of SDPP.



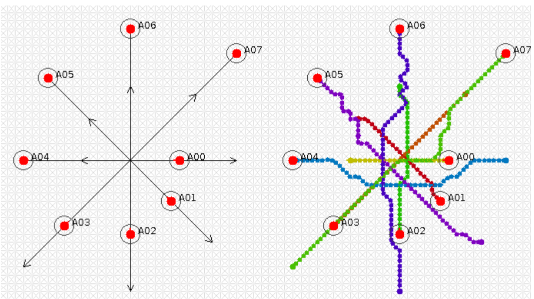
(a) Single superconflict scenario example.



(b) Four homogenous superconflicts scenario example.



(c) Four heterogeneous superconflicts scenario example.



(d) Spiral superconflict scenario example.

Figure 5: Superconflict scenarios example – problem configurations (left) and solutions from IADPP algorithm (right).

	CA	SDPP	ADPP	IADPP
single superconflict	10.30 s	26.24 s	11.91 s	9.50 s
four homogeneous superconflicts	45.81 s	26.97 s	13.86 s	11.62 s
four heterogeneous superconflicts	9.084 s	16.01 s	4.89 s	2.59 s
spiral superconflict	6.15 s	21.02 s	17.64 s	3.77 s

Table 1: Wall-clock runtimes for four versions of superconflict scenario (averaged over 10 runs)

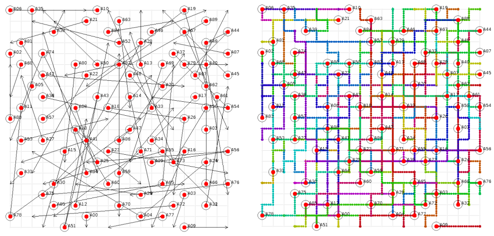


Figure 6: One instance of random scenario with 90 agents. The start and goal position of each agent are depicted on the left, the final solution found is on the right.

Random scenario

We measured the wall-clock runtime, communication complexity and solution quality of the four algorithms CA, SDPP, ADPP and IADPP on a series of problem instances that varied in the number of agents from 30 to 100. The start and goal vertices for each agent in the scenario were selected randomly (see Figure 6). The distance between the start and goal position was taken uniformly from the interval $(5,10)$ and we further asserted that no two agents share the start node and no two agents share the destination node. The agents plan their trajectory on a 20x20 4-connected grid graph. For each number of agents we ran 10 different random scenarios and averaged the results. When any of the algorithms failed to find a solution to a problem instance, the problem instance was excluded from the experiment.

The wall-clock runtime represents the real-world time a particular algorithm would need to converge to a solution. The wall-clock time for CA is equal to its CPU-time and can be measured directly. The average wall-clock runtime of the three decentralized algorithms on random scenarios with n agents was obtained by running an n concurrent processes simulation of the algorithm execution. The results for the wall-clock runtime experiment are shown in Figure 7a. We can see that all decentralized algorithms can offer a speed-up over the centralized solver. Further, we find that ADPP and IADPP provide comparable wall-clock runtime performance, which is significantly better than the runtime performance of SDPP, especially in dense problem instances with many conflicting agents.

Further, we measured the communication complexity by counting the messages each of the algorithms broadcasts during the execution. The communication complexity of the CA algorithm is computed analytically. We assume that the algorithm is used to coordinate paths in a distributed system in the following way. All the agents are required to communicate their objectives to the central solver. When the central solver finishes the planning, it informs each agent about its new path. Thus, we use $2n$ as the communication complexity of the centralized solver. In Figure 7b we can see that the decentralized algorithms start exceeding the communication complexity of the centralized solution for scenarios with more than 60 agents. Further, we find that IADPP algorithm has lower communication complexity than ADPP. This can be explained by looking at how the two algorithms react to an inform message that invalidates the current running planning effort. In ADPP, the planning is finished, the new plan broadcast and only after that a new planning is started. In IADPP, the planner is restarted quietly, yielding no extra communication.

Figure 7c shows the quality of the generated solutions. The reason why decentralized algorithms return on average slightly worse solutions than the CA algorithm lies in the replanning condition used by the decentralized algorithms. The condition states that an agent should replan his trajectory only if the trajectory is inconsistent with his agentview. In result, the agent may receive an updated trajectory from a higher-priority agent that allows for improvement in his current trajectory, but since the trajectory may be still consistent, the agent will not exploit such an improvement opportunity.

Finally, Figure 7d shows the failure rates of the individual algorithms as a function of the number of agents in a scenario.

6 Conclusion

In this paper we introduced an asynchronous decentralized prioritized planning algorithm for space-time cooperative pathfinding problem. Two variants of the algorithm, ADPP and IADPP, were presented. We proved the correctness and termination of both introduced algorithms. The algorithms were compared to both central and decentralized state-of-the-art techniques for prioritized planning. Experimental validation and evaluation showed the benefits and limitations of the discussed algorithms. The experiments show the advantages of asynchronous and interruptible execution of the presented algorithms on a set of superconflict scenarios.

The large scale evaluation on a set of random problem instances documents a significant reduction of average wall-clock runtime of both ADPP and IADPP in comparison to the centralized (approx. 65% time reduction) and the decentralized synchronous algorithm (approx. 45% time reduction). The communication complexity is the worst for ADPP, while IADPP is still better than SDPP, but worse than CA for higher numbers of agents. The average cost of generated solutions is similar for all decentralized algorithms and only approx. 10% worse than CA. The failure ratio of all prioritized methods is comparable. The experimental validation fully supports the expectations on the improvements of the ADPP and IADPP over both CA and SDPP.

Acknowledgements

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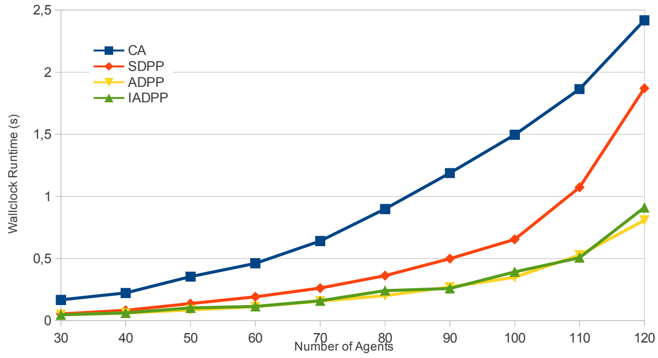
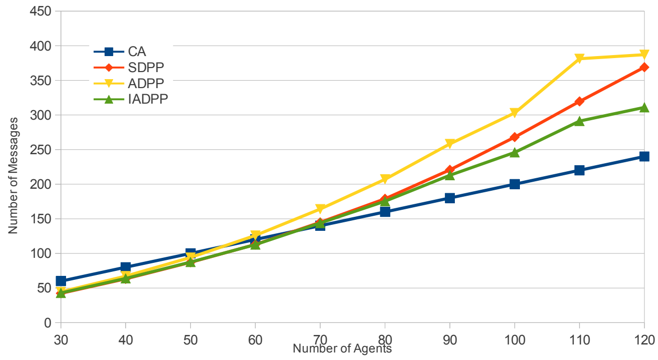
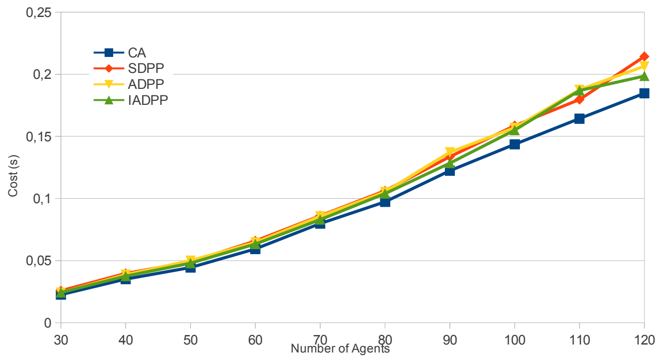


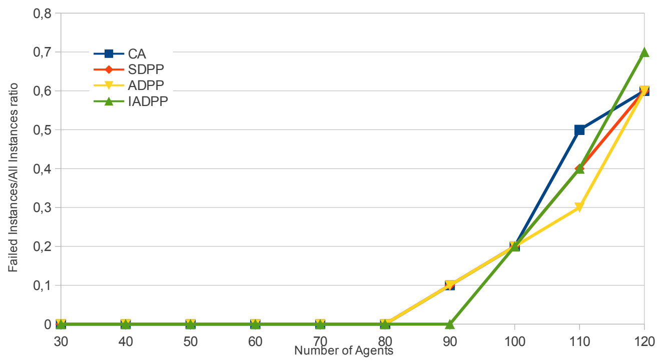
(a) Average wall-clock runtime for *n*-agent random scenario



(b) Average messages broadcasted



(c) Average cost (prolongation of trajectories)



(d) Failed instances ratio

Figure 7: Results from the random scenario